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Abelian Family Symmetries and Leptogenesis

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Abstract

We study the impact of a set of horizontal symmetries on the requirements for producing the baryon asymmetry of the universe via leptogenesis. We find that Abelian horizontal symmetries lead to a simple description of the parameters describing leptogenesis in terms of the small expansion parameter that arises from spontaneous symmetry breaking. If the family symmetry is made discrete, then an enhancement in the amount of leptogenesis can result.

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I. INTRODUCTION

There is now strong evidence for atmospheric neutrino oscillations. The data suggests [1] that $\nu_\mu - \nu_\tau$ oscillations occur with near maximal mixing $\sin^2 2\theta_{23} \approx 1$ and a mass splitting of $\Delta m^2_{23} \sim 2.2 \times 10^{-3}$ eV^2. The measured solar neutrino flux can be explained by oscillations of ν_e to the other two generations ($x = 2, 3$). In the case of matter oscillations (MSW) there are two solutions: (1) the small mixing angle (SMA) solution for which $\Delta m^2_{1x} \sim 5 \times 10^{-6}$ eV^2 and $\sin^2 2\theta_{1x} \sim 6 \times 10^{-3}$, and (2) the large mixing angle (LMA) solution for which $\Delta m^2_{1x} \sim 2 \times 10^{-5}$ eV^2 and $\sin^2 2\theta_{1x} \sim 0.8$. In the case of vacuum oscillations (VO) the mass-squared difference is much smaller $\Delta m^2_{1x} \sim 8 \times 10^{-11}$ eV^2 and the mixing angle is also large, $\sin^2 2\theta_{1x} \sim 0.8$. The largeness of the mixing θ_{23} and possibly in θ_{1x} and the apparent hierarchy in the associated masses presents something of a dilemma, since one would expect that large mixing of order one occur when the eigenvalues (neutrino masses) are roughly degenerate. Many models have been proposed to account for the neutrino oscillation data, and it is interesting to explore whether these models can account in a natural way for the baryon asymmetry of the universe through the process of leptogenesis. In this paper we explore the implications for Abelian family symmetries on lepton asymmetries generated in the early universe. In particular we argue that a discrete Z_2 component can not only resolve the dilemma of large mixing together with a large hierarchy mentioned above, but it can also lead to an enhanced baryon asymmetry.

II. THE BARYON ASYMMETRY AND LEPTOGENESIS

The lightness of the three known neutrinos can be understood as arising from the see-saw mechanism where right-handed neutrinos, being Standard Model singlets, have a very large mass. The addition of right-handed singlet neutrinos to the Standard Model leads to lepton number violation. The existence of very heavy right-handed neutrinos are predicted by grand unified theories based on the gauge group $SO(10)$, and the lightness of the observed neutrinos can be explained via a see-saw mechanism. Since the heavy right-handed neutrinos offer a reasonable basis for the observed oscillations and neutrino masses, it motivates the consideration of their possible cosmological effects. Since these particles would naturally occur in the early universe, it is of interest to determine whether it is possible that the decays of these heavy particles could be the source of the baryon asymmetry of the universe [2].

The nonzero net baryon density $n_B - n_{\overline{B}}$ of the universe can be accounted for in theories that satisfy Sakharov's conditions [3]: 1) baryon number is violated, 2) charge conjugation symmetry (C) and CP are violated, and 3) there is a departure from thermal equilibrium. A nontrivial requirement on any particle theory satisfying these three conditions is that a sufficient asymmetry in n_B and $n_{\overline{B}}$ be produced to explain the observed value of the ratio of net baryon density to the entropy density s of the universe

$$Y_B = \frac{n_B - n_{\overline{B}}}{s} = (0.6 - 1) \times 10^{-10} \; . \tag{1}$$

The Standard Model in the early universe satisfies all three conditions, but it is generally agreed that the produced asymmetry is too small [4]. Therefore one is motivated to look

beyond the Standard Model at theories that contain new sources of baryon number violation and CP-violation and/or for theories that have a new mechanism for producing the asymmetry. If one instead considers the Minimal Supersymmetric Standard Model (MSSM) then the regions of parameter space where sufficient baryon asymmetry is produced is quite small [5]. Consequently various proposals have been made for new physics capable of producing the baryon asymmetry of the universe. One of the most attractive of these is the possibility that CP violating decays of heavy neutrinos can produce an excess of leptons over antileptons (or vice versa). The lepton asymmetry produced in the early universe via out-of-equilibrium decays of the right-handed neutrinos is subsequently recycled into a baryon asymmetry by sphaleron transitions (which violated both baryon number and lepton number). A straightforward analysis of chemical potentials for equilibrating processes including the sphaleron transition relates the baryon asymmetry Y_B to the original lepton asymmetry $Y_L = (n_L - n_{\overline{L}})/s$ via [6,7]

$$Y_B = aY_{B-L} = \frac{a}{a-1}Y_L \ , \qquad a = \frac{8N_F + 4N_H}{22N_F + 13N_H} \ , \tag{2}$$

where N_F is the number of fermion families and N_H is the number of Higgs doublets. So the final baryon asymmetry present in the universe today is related to the lepton asymmetry Y_L by an order one parameter. If one accepts the presence of heavy Majorana neutrinos in nature, then CP-violation naturally occurs and the question becomes whether or not the lepton asymmetry that results is the right order of magnitude for producing the observed baryon asymmetry in Eq. (1). In the MSSM with heavy right-handed neutrinos, the resulting lepton asymmetry has been shown to be sufficient to explain the observed baryon asymmetry in a natural way in a number of models [8–11].

Most work in trying to understand the structure of the fermion masses and mixings has tried to fit the low energy data, e.g. the fermion masses and the CKM matrix as well as the neutrino data (especially the solar neutrino oscillation data and the atmospheric neutrino oscillation data). If one accepts the notion that leptogenesis is the source of the baryon asymmetry of the universe, then this mechanism imposes another rather strong constraint on the details of the family symmetry (this kind of symmetry is also called a horizontal symmetry). For example the lepton asymmetry produced by the decay of heavy Majorana neutrinos is sensitive to the texture pattern of the Yukawa matrices as well as the details of the mass and mixing hierarchies [12]. In the next section we apply the strategy of employing an Abelian family symmetry to describe the hierarchies and discuss the implications for leptogenesis.

III. HORIZONTAL SYMMETRIES

One attempt at accounting for the fermion mass spectrum makes use of broken family symmetries [13]. The most common approach is to take an Abelian $U(1)$ as the horizontal symmetry, but nonabelian groups and discrete groups (and combinations of these) have been tried with varying degrees of success. Since an Abelian symmetry alone cannot generate a nearly degenerate set of neutrinos [17], we assume here that the Δm^2_{23} and Δm^2_{1x} are indicating that the neutrino masses are arranged in a hierarchical pattern. This hierarchical

structure of the fermion masses suggests that it might be produced by an expansion in a small parameter, and one widely adopted strategy is to have this parameter arise from a family symmetry spontaneously broken at a scale Λ_L. In this paper we consider the possibility that the horizontal symmetry is an Abelian anomalous gauge symmetry [14,15], where the anomaly is cancelled by the Green-Schwarz mechanism [16]. In this scenario there is field Φ that is a singlet under the Standard Model gauge symmetries. The contribution of the Fayet-Iliopoulos term to the D-term cancels against the contribution from the vev, $< \Phi >$. The ratio of this vev to the Planck scale naturally provides a small parameter $\lambda =< S > / m_{\rm Pl}$. The field Φ is charged under the horizontal symmetry, and without loss of generality it charge can be taken to be -1. In this approach the hierarchy is generated by nonrenormalizable terms that transform as singlets under the horizontal symmetry and therefore produce contributions to the mass matrices that contain integer powers of the small parameter λ. In this scenario it is often the case that only the $(3,3)$ entry of one or more mass matrices receives a contribution from a renormalizable coupling to the Higgs boson. So by assigning quantum numbers for the horizontal symmetry for each Standard Model field, one can generate a hierarchy in the Yukawa matrices as powers of the small parameter λ.

The heavy Majorana neutrino mass matrix M_N is obtained by inverting the type-I see-saw formula

$$m_\nu = m_D (M_N)^{-1} m_D^T \;, \tag{3}$$

where m_D is the neutrino Dirac mass matrix. CP-asymmetries in neutrino decays arise from the interference between the tree level and one-loop level decay channels. In the mass basis where the right-handed Majorana mass matrix is diagonal the asymmetry in heavy neutrino N_i decays

$$\epsilon_i = \frac{\Gamma(N_i \to \ell H_2) - \Gamma(N_i \to \ell^c H_2^c)}{\Gamma(N_i \to \ell H_2) + \Gamma(N_i \to \ell^c H_2^c)} \;, \tag{4}$$

is given by [8,18]

$$\epsilon_i = \frac{3}{16\pi v_2^2} \frac{1}{(m_D^\dagger m_D)_{ii}} \sum_{n \neq i} \mathrm{Im}\left[(m_D^\dagger m_D)^2_{ni} \right] \frac{M_i}{M_n} \;. \tag{5}$$

The masses M_i are the three eigenvalues of the heavy Majorana mass matrix and v_2 is the vev of the Higgs giving Dirac masses to the neutrinos and up-type quarks. M_1 is the mass of the lightest of the three heavy Majorana neutrinos, and Eq. (5) is an approximate formula valid for $M_n >> M_i$. The most common scenario that occurs is that the lightest Majorana neutrino N_1 has a mass such that $M_1 << M_2, M_3$, and the lepton asymmetry produced[1] comes almost entirely from the decays of N_1. So the CP-asymmetry of most interest to the discussion of lepton asymmetry generation is ϵ_1.

The other parameter of most interest is the mass parameter

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[1] In some cases inverted hierarchies in the Majorana mass matrix can occur where $M_2 < M_1$, which can produce a larger asymmetry if $\epsilon_2 > \epsilon_1$ [19]. We do not consider this possibility in this paper.

$$\tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} \,, \tag{6}$$

which controls the decay width of the lightest right-handed neutrino N_1 since

$$\Gamma_{N_i} = \Gamma(N_i \to \ell H_2) + \Gamma(N_i \to \ell^c H_2^c) = \frac{1}{8\pi}(m_D^\dagger m_D)_{ii}\frac{M_i}{v_2^2} \,, \tag{7}$$

and $\tilde{m}_1$ also largely controls the amount of dilution caused by the lepton number violating scattering. The parameter $\tilde{m}_1$ can therefore be called the dilution mass. These two constraints bound the possible values of $\tilde{m}_1$ such that a sufficient asymmetry is produced to agree with Eq. (1). The generated lepton asymmetry is given by

$$Y_L = \frac{n_L - n_{\overline{L}}}{s} = \kappa\frac{\epsilon_1}{g^*} \,, \tag{8}$$

where g^* is the number of light (effective) degrees of freedom in the theory ($106\,\frac{3}{4}$ in the Standard Model or $228\,\frac{3}{4}$ in the MSSM), and κ is a dilution factor that can be reliably calculated by solving the full Boltzmann equations. The dilution depends critically on the parameter $\tilde{m}_1$ because it governs the size of the most important Yukawa coupling in the $\Delta L = 2$ scattering processes, as shown in Ref. [8].

A. Leptogenesis with a $U(1)$ family symmetry

Assume now that the lepton fields have charges under a $U(1)$ family symmetry

$$\begin{array}{ccccccccc} e^c_{R1} & e^c_{R2} & e^c_{R3} & \ell_{L1} & \ell_{L2} & \ell_{L3} & \nu^c_{R1} & \nu^c_{R2} & \nu^c_{R3} \\ E_1 & E_2 & E_3 & L_1 & L_2 & L_3 & \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 \end{array} .$$

We assume here that the quantum numbers satisfy the hierarchies $E_1 \geq E_2 \geq E_3 \geq 0$, $L_1 \geq L_2 \geq L_3 \geq 0$, and $\mathcal{N}_1 \geq \mathcal{N}_2 \geq \mathcal{N}_3 \geq 0$ (This last condition will guarantee that no light neutrino masses are enhanced because a right-handed neutrino mass is suppressed [20–22]).

Given lepton doublet charges L_i and right-handed neutrino charges $\mathcal{N}_i$ one has the following pattern for the neutrino Dirac mass matrix

$$m_D \sim \begin{pmatrix} \lambda^{L_1+\mathcal{N}_1} & \lambda^{L_1+\mathcal{N}_2} & \lambda^{L_1+\mathcal{N}_3} \\ \lambda^{L_2+\mathcal{N}_1} & \lambda^{L_2+\mathcal{N}_2} & \lambda^{L_2+\mathcal{N}_3} \\ \lambda^{L_3+\mathcal{N}_1} & \lambda^{L_3+\mathcal{N}_2} & \lambda^{L_3+\mathcal{N}_3} \end{pmatrix} v_2 \,, \tag{9}$$

and the following pattern for the Majorana mass matrix

$$M_N \sim \begin{pmatrix} \lambda^{2\mathcal{N}_1} & \lambda^{\mathcal{N}_1+\mathcal{N}_2} & \lambda^{\mathcal{N}_1+\mathcal{N}_3} \\ \lambda^{\mathcal{N}_1+\mathcal{N}_2} & \lambda^{2\mathcal{N}_2} & \lambda^{\mathcal{N}_2+\mathcal{N}_3} \\ \lambda^{\mathcal{N}_1+\mathcal{N}_3} & \lambda^{\mathcal{N}_2+\mathcal{N}_3} & \lambda^{2\mathcal{N}_3} \end{pmatrix} \Lambda_L \,. \tag{10}$$

Then one obtains the following form for the light neutrino mass matrix via the see-saw formula Eq. (3)

$$m_\nu \sim \begin{pmatrix} \lambda^{2L_1} & \lambda^{L_1+L_2} & \lambda^{L_1+L_3} \\ \lambda^{L_1+L_2} & \lambda^{2L_2} & \lambda^{L_2+L_3} \\ \lambda^{L_1+L_3} & \lambda^{L_2+L_3} & \lambda^{2L_3} \end{pmatrix} \frac{v_2^2}{\Lambda_L} , \tag{11}$$

Clearly if $L_2 = L_3$ one can obtain $\mathcal{O}(1)$ mixing in the 2-3 sector [23], or if $L_2 = -L_3$ one has a pseudo-Dirac neutrino and maximal mixing in the 2-3 sector [24].[2]

The Super-Kamiokande collaboration measurements of the atmospheric neutrino flux indicates large mixing $\sin^2 2\theta_{23} \sim 1$ and a mass-squared difference $\Delta m^2_{23} \sim 2\times 10^{-3}$ eV^2. The SMA solution to the solar neutrino oscillations requires $\Delta m^2_{1x} \sim 5\times 10^{-6}$ eV^2. If one assumes that the light neutrino masses are hierarchical, then one can identify $m^2_{\nu_\tau} \sim 2 \times 10^{-3}$ eV^2 and $m^2_{\nu_\mu} \sim 5 \times 10^{-6}$ eV^2; it is then difficult to naturally explain the separation of masses simultaneously with the large mixing angle. The suppression of one of the neutrino masses can always result from a fine-tuning of the parameters.

The dilution parameter $\tilde{m}_1$ defined in Eq. (6) can be described in terms of the $U(1)$ quantum numbers by constructing the Yukawa coupling squared matrix

$$m_D^\dagger m_D \sim \begin{pmatrix} \lambda^{2\mathcal{N}_1} & \lambda^{\mathcal{N}_1+\mathcal{N}_2} & \lambda^{\mathcal{N}_1+\mathcal{N}_3} \\ \lambda^{\mathcal{N}_1+\mathcal{N}_2} & \lambda^{2\mathcal{N}_2} & \lambda^{\mathcal{N}_2+\mathcal{N}_3} \\ \lambda^{\mathcal{N}_1+\mathcal{N}_3} & \lambda^{\mathcal{N}_2+\mathcal{N}_3} & \lambda^{2\mathcal{N}_3} \end{pmatrix} \lambda^{2L_3} v_2^2 , \tag{12}$$

so that

$$\tilde{m}_1 \sim \frac{\lambda^{2(L_3+\mathcal{N}_1)} v_2^2}{M_1} \sim \frac{\lambda^{2(L_3+\mathcal{N}_1)}}{\lambda^{2\mathcal{N}_1}} \frac{v_2^2}{\Lambda_L} \sim \lambda^{2L_3} \frac{v_2^2}{\Lambda_L} , \tag{13}$$

When $L_2 = L_3$ then this parameter is the same order of magnitude as the neutrino masses m_{ν_μ} and m_{ν_τ}[3], and it is consistent to take the parameter $\tilde{m}_1 \sim (m_{\nu_\mu} m_{\nu_\tau})^{1/2}$. Typically one needs a fine-tuning to produce the hierarchy $m_{\nu_\mu} << m_{\nu_\tau}$. The CP-violating parameter is given by

$$\epsilon_1 \sim \frac{3}{16\pi} \lambda^{2(L_3+\mathcal{N}_1)} . \tag{14}$$

Comparing to Eq. (13), one sees that ϵ_1 can be simply expressed in terms of the dilution mass $\tilde{m}_1$, the mass M_1 of the lightest Majorana neutrino, and the electroweak scale vev v_2. Since $\tilde{m}_1$ is tied to the light neutrino masses, a connection between these quantities is established at the order-of-magnitude level.

The problem with the situation outlined is well-known: it seems to predict that m_{ν_μ} is the naturally of the same order as m_{ν_τ}, and one would need to have an accidental cancellation to get the hierarchy $m_{\nu_\mu} << m_{\nu_\tau}$. The charged lepton matrix is given by

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[2]It is also possible that one has only an approximate equality $L_2 \approx \pm L_3$ in which case the mixing is not truly order one, but could be sufficiently large to be phenomenologically relevant without assuming accidental cancellations [21].

[3]We use the notation ν_μ and ν_τ for the eigenstates even though they have large mixing.

$$m_{\ell^\pm} \sim \begin{pmatrix} \lambda^{L_1+E_1} & \lambda^{L_1+E_2} & \lambda^{L_1+E_3} \\ \lambda^{L_2+E_1} & \lambda^{L_2+E_2} & \lambda^{L_2+E_3} \\ \lambda^{L_3+E_1} & \lambda^{L_3+E_2} & \lambda^{L_3+E_3} \end{pmatrix} v_1 \ , \tag{15}$$

where v_1 is the vev of the other Higgs doublet. So the relevant rotation to get to the basis where the charged lepton mass is diagonal is also order one when $L_2 = L_3$. Hence the large mixing in the 2-3 sector is connected in this approach to near degeneracy of two of the light neutrino masses.

B. Leptogenesis with a $Z_2 \times U(1)$ family symmetry

Ref. [20] proposed that a discrete Abelian family symmetry could be employed to enhance a mass or mixing angle above what would be otherwise obtained if the family symmetry was the usual continuous $U(1)$ symmetry, and this idea was pursued further in a specific model [25]. If the family symmetry is Z_m then entries in the mass matrices can be enhanced by factors of the small parameter λ to the mth power. With this approach the discrete Z_m symmetry can result in the enhancement of entries in the light neutrino mass matrix. A consequence for leptogenesis is that this will also change the relationship between the light neutrino masses and the dilution parameter $\tilde{m}_1$ by a factor of λ^m. For example take the following $Z_2 \times U(1)$ charges for the lepton fields[4]

e^c_{R1}	e^c_{R2}	e^c_{R3}	ℓ_{L1}	ℓ_{L2}	ℓ_{L3}	ν^c_{R1}	ν^c_{R2}	ν^c_{R3}
$(0, E_1)$	$(0, E_2)$	$(0, E_3)$	$(0, L_1)$	$(0, L_2)$	$(1, L_3 - 1)$	$(0, \mathcal{N}_1)$	$(0, \mathcal{N}_2)$	$(1, \mathcal{N}_3 - 1)$

and later we will take $L_2 = L_3$. Assume the symmetry breaking is characterized by the single expansion parameter λ. The formulas given above for the heavy neutrino mass matrix, M_N, the neutrino Dirac mass matrix, m_D, and the resulting light neutrino mass matrix, m_ν are modified. With the above assignments one finds that

$$M_N \sim \begin{pmatrix} \lambda^{2\mathcal{N}_1} & \lambda^{\mathcal{N}_1+\mathcal{N}_2} & \lambda^{\mathcal{N}_1+\mathcal{N}_3} \\ \lambda^{\mathcal{N}_1+\mathcal{N}_2} & \lambda^{2\mathcal{N}_2} & \lambda^{\mathcal{N}_2+\mathcal{N}_3} \\ \lambda^{\mathcal{N}_1+\mathcal{N}_3} & \lambda^{\mathcal{N}_2+\mathcal{N}_3} & \lambda^{2\mathcal{N}_3-2} \end{pmatrix} \Lambda_L \ , \tag{16}$$

so that

$$(M_N)^{-1} \sim \begin{pmatrix} \lambda^{-2\mathcal{N}_1} & \lambda^{-\mathcal{N}_1-\mathcal{N}_2} & \lambda^{-\mathcal{N}_1-\mathcal{N}_3+2} \\ \lambda^{-\mathcal{N}_1-\mathcal{N}_2} & \lambda^{-2\mathcal{N}_2} & \lambda^{-\mathcal{N}_2-\mathcal{N}_3+2} \\ \lambda^{-\mathcal{N}_1-\mathcal{N}_3+2} & \lambda^{-\mathcal{N}_2-\mathcal{N}_3+2} & \lambda^{-2\mathcal{N}_3+2} \end{pmatrix} \Lambda_L^{-1} \ . \tag{17}$$

Furthermore one has

$$m_D \sim \begin{pmatrix} \lambda^{L_1+\mathcal{N}_1} & \lambda^{L_1+\mathcal{N}_2} & \lambda^{L_1+\mathcal{N}_3} \\ \lambda^{L_2+\mathcal{N}_1} & \lambda^{L_2+\mathcal{N}_2} & \lambda^{L_2+\mathcal{N}_3} \\ \lambda^{L_3+\mathcal{N}_1} & \lambda^{L_3+\mathcal{N}_2} & \lambda^{L_3+\mathcal{N}_3-2} \end{pmatrix} v_2 \ , \tag{18}$$

[4] The second group factor does not need to be continuous, but could be replaced by a second Z_n with n sufficiently large.

Then it is straightforward to show that the light neutrino mass matrix m_ν is modified so that only one component is enhanced,

$$m_\nu \sim \begin{pmatrix} \lambda^{2L_1} & \lambda^{L_1+L_2} & \lambda^{L_1+L_3} \\ \lambda^{L_1+L_2} & \lambda^{2L_2} & \lambda^{L_2+L_3} \\ \lambda^{L_1+L_3} & \lambda^{L_2+L_3} & \lambda^{2L_3-2} \end{pmatrix} \frac{v_2^2}{\Lambda_L} \,. \tag{19}$$

Also one finds that

$$m_D^\dagger m_D \sim \begin{pmatrix} \lambda^{2\mathcal{N}_1} & \lambda^{\mathcal{N}_1+\mathcal{N}_2} & \lambda^{\mathcal{N}_1+\mathcal{N}_3-2} \\ \lambda^{\mathcal{N}_1+\mathcal{N}_2} & \lambda^{2\mathcal{N}_2} & \lambda^{\mathcal{N}_2+\mathcal{N}_3-2} \\ \lambda^{\mathcal{N}_1+\mathcal{N}_3-2} & \lambda^{\mathcal{N}_2+\mathcal{N}_3-2} & \lambda^{2\mathcal{N}_3-4} \end{pmatrix} \lambda^{2L_3} v_2^2 \,. \tag{20}$$

So then using our previous definitions, one sees that $\tilde{m}_1 \sim \lambda^{2L_3} v_2^2/\Lambda_L \sim m_{\nu_\mu}$, whereas $m_{\nu_\tau} \sim \lambda^{2L_3-2} v_2^2/\Lambda_L$. More specifically when the atmospheric neutrino constraint $\Delta m^2_{23} \sim 2 \times 10^{-3}$ eV^2 is interpreted as the mass-squared of the heaviest light neutrino m_{ν_τ}, then in the case of a horizontal $U(1)$ symmetry, one has that $\tilde{m}_1^2 \sim 2 \times 10^{-3}$ eV^2. In the case of the discrete Z_2 symmetry, the Yukawa coupling related to $\tilde{m}_1^2$ via Eq. (6) can be reduced by a factor λ^2 thereby substantially reducing the amount of dilution from the $\Delta L = 2$ processes and reducing the decay rate of N_1. More generally, a Z_m symmetry can arrange for a suppression of $\tilde{m}_1^2$ by a factor λ^m. The CP-violation asymmetry ϵ_1 is easily obtained from Eqs. (5) and (20),

$$\epsilon_1 \sim \frac{3}{16\pi} \lambda^{2(L_3+\mathcal{N}_1)-2} \,. \tag{21}$$

So the ratio of m_{ν_τ} to ϵ_1 is unaffected by the discrete symmetry. Since m_{ν_τ} is being fixed by the experimental data for Δm^2_{23}, the expected value of ϵ_1 is expected to be unchanged for the same quantum number $\mathcal{N}_1$ (and thus the same heavy neutrino mass M_1).

A phenomenologically viable solution has been presented in Ref. [20]: taking $L_1 = 3$, $L_2 = L_3 = 1$, $E_1 = 5$, $E_2 = 4$, and $E_3 = 2$ yields mass matrices of the form

$$m_\nu \sim \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \frac{v_2^2}{\Lambda_L} \,, \qquad m_{\ell^\pm} \sim \begin{pmatrix} \lambda^8 & \lambda^7 & \lambda^5 \\ \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^6 & \lambda^5 & \lambda^3 \end{pmatrix} v_1 \,, \tag{22}$$

which give the correct order of magnitude for the SMA solution (after rotating to the charged lepton mass basis)

$$\frac{\Delta m^2_{1x}}{\Delta m^2_{23}} \sim \lambda^4, \quad \sin\theta_{12} \sim \lambda^2, \quad \sin\theta_{23} \sim 1, \quad \sin\theta_{13} \sim \lambda^2, \tag{23}$$

when the small parameter is identified as the Cabibbo angle, i.e. $\lambda \sim 0.2$.

For a sufficient amount of leptogenesis to occur two conditions must be satisfied: (1) $|\epsilon_1| \gtrsim 10^{-6}$ and (2) $10^{-5} \lesssim \tilde{m}_1 \lesssim 10^{-2}$. The first condition guarantees that there is sufficient CP-violation in the heavy N_1 neutrino decay (c.f. Eq. (8), while the second condition guarantees that the dilution is not too large ($\kappa \gtrsim 10^{-2}$) and that a sufficient number of heavy neutrino are produced out-of-equilibrium [9]. The condition on $\tilde{m}_1$ is equivalent to a condition on

the relevant Yukawa coupling $(h_\nu^\dagger h_\nu)_{11} = m_D^\dagger m_D/v_2^2$ that governs the rates of these two processes.

The mass $\tilde{m}_1$ arising in the case of the $U(1)$ symmetry is identified with m_{ν_τ}, and thus is near the top of the required range. The resulting lepton asymmetry is smaller than it would be if the dilution mass $\tilde{m}_1$ could be reduced. Lowering the mass parameter $\tilde{m}_1$ by using the horizontal $Z_2 \times U(1)$ rather than the $U(1)$ symmetry has the following effects on the Boltzmann evolution: 1) The lightest Majorana neutrino N_1 decays more slowly, and stays out of thermal equilibrium for a longer period of time. 2) The dilution of the generated lepton asymmetry is reduced since the relevant Yukawa coupling controlling the strength of the interactions is reduced. These two factors can result in a remnant lepton asymmetry that is enhanced over that which is obtained in the case of the $U(1)$ symmetry.

IV. NUMERICAL SIMULATION

The lepton asymmetry that results can be obtained by integrating the full set of Boltzmann equations [26]. These differential equations, incorporating the Majorana neutrino decay rates as well as all lepton number violating scattering processes in the MSSM, has been given in Ref. [9]. The above discussion gives an overall order of magnitude estimate for the CP-violation parameter ϵ_1 and the dilution parameter $\tilde{m}_1$. The parameter ϵ_1 depends on a CP-phase (see Eq. (5)); this phase is not specified by the family symmetry and we assume that it is order one.

A concrete example of how the discrete symmetry can change the produced lepton asymmetry is shown in Figs. 1 and 2. First consider the case where the family symmetry is $U(1)$: the values of the parameters are $(m_D^\dagger m_D)_{11} = 0.2$ GeV^2 and $\epsilon_1 = -4.0 \times 10^{-6}$ for the case of the continuous $U(1)$ symmetry. The values for the CP-violation parameter ϵ_1 in Eq. (14) and $(m_D^\dagger m_D)_{11} \sim \lambda^{2(L_3+\mathcal{N}_1)} v_2^2$ are consistent[5] with taking $L_3 = 0$ and $\mathcal{N}_1 = 3$. Taking the scale Λ_L to be near a supersymmetric grand unified scale $\sim 3\times10^{15}$ GeV, one finds the mass of the lightest heavy neutrino N_1 that is decaying asymmetrically to be $M_1 = 2\times10^{11}$ GeV. This then yields a dilution mass of $\tilde{m}_1 = 5\times10^{-3}$ eV, which is the same order of magnitude as the mass splitting $\Delta m_{23}^2 \sim 2.2 \times 10^{-3}$ eV^2 as expected from Eqs. (11) and (13).

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[5] These relationships involving ϵ_1 and $(m_D^\dagger m_D)_{11}$ only determine the leading order contribution in the small parameter λ and there is an undetermined coefficient of order one. We choose the values of the parameters here as an example to illustrate that an enhancement occurs. The exact values the unknown order one coefficients take are not important; the enhancement of the lepton asymmetry is generic.

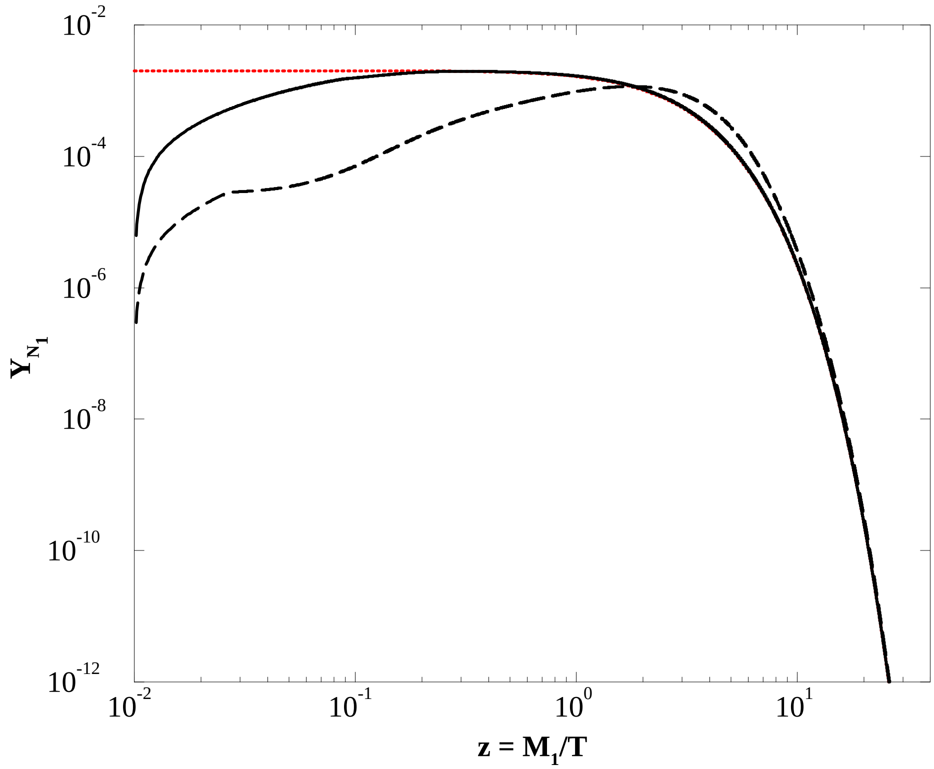

Fig. 1. The neutrino density Y_{N_1} as a function of the temperature T of the universe for the case of a horizontal $U(1)$ symmetry (solid), and for the $Z_2 \times U(1)$ symmetry (dashed). The dotted curve is the equilibrium value $Y_{N_1}^{\rm eq}$ of the neutrino density. The discrete symmetry results in a smaller decay rate for N_1 and it requires a longer time before it comes into thermal equilibrium.

When the $U(1)$ symmetry is replaced with Z_2 the dilution mass is suppressed by an additional factor of λ^2 so that $\tilde{m}_1 = 2.2 \times 10^{-4}$ eV (for the case of the $Z_2 \times U(1)$ symmetry, we take $L_3 = 1$ and keep $\mathcal{N}_1 = 3$ so that m_{ν_τ} and ϵ_1 remain the same, but $\tilde{m}_1$ is reduced by a factor λ^2 relative to the $U(1)$ symmetry case.). Figure 1 shows the neutrino density Y_{N_1} of the lightest Majorana neutrino that is decaying to produce to produce the lepton asymmetry shown in Fig. 2. The densities are plotted against the dimensionless ratio $z = M_1/T$ where T is the temperature of the universe, so the universe evolves toward the present day as z becomes larger. For the quantitative results shown in the figures, the unknown CP phase (see Eq. (5) is chosen so as to maximize the lepton asymmetry; another phase would just scale the curves in Fig. 2 by some overall factor. The Z_2 symmetry results in N_1 decaying more slowly, and thus N_1 can remain out-of-equilibrium for a greater period of time in the early universe. The lepton asymmetry produced in each case begins with one sign, then goes through zero, and finally asymptotes to a final value. For the particular example shown in Figs. 1 and 2, the lepton asymmetry is enhanced by about a factor between seven and eight when the continuous family symmetry is replaced by a discrete one. The enhancement (or suppression) that can result in general (from suppressing $\tilde{m}_1$) is a sensitive function of the values of dilution parameter $\tilde{m}_1$ and the mass M_1, as shown in Ref. [9].

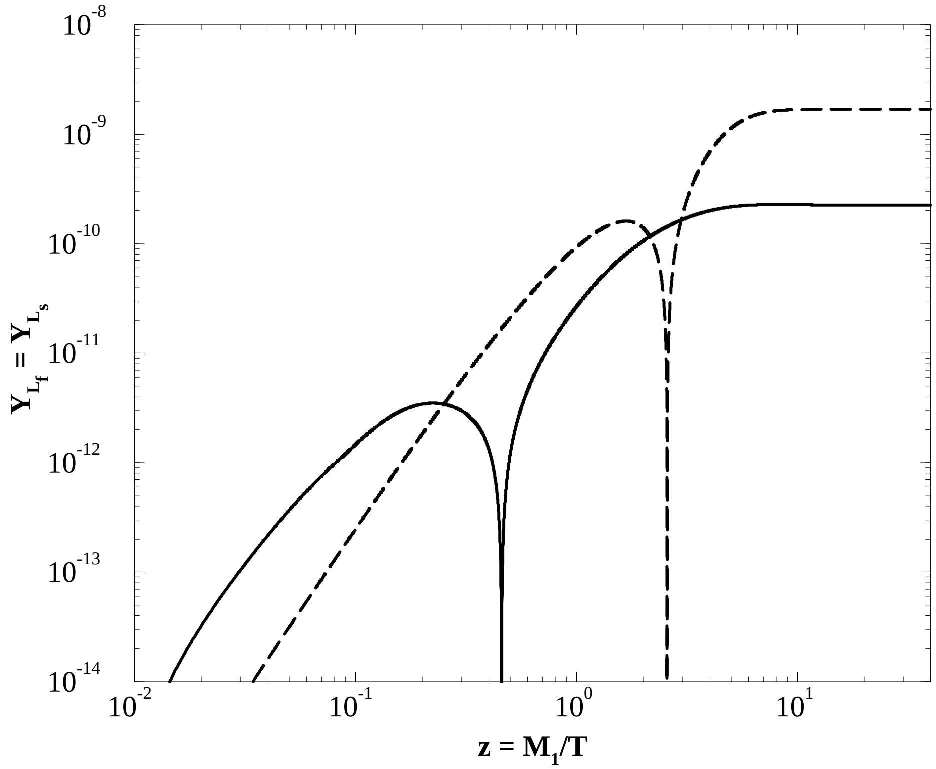

Fig. 2. The lepton asymmetry in fermions Y_{L_f} and in scalars Y_{L_s} produced for a horizontal $U(1)$ symmetry (solid), and for the $Z_2 \times U(1)$ symmetry (dashed). The generated asymmetry in the latter case is smaller at earlier times (larger temperatures) since the decay rate of the lightest Majorana neutrino N_1 is suppressed, but ultimately a larger asymmetry is produced as the neutrino density remains out of thermal equilibrium for a longer period. The equality $Y_{L_f} = Y_{L_s}$ is maintained by MSSM processes $f + f \leftrightarrow \tilde{f} + \tilde{f}$, e.g. neutralino exchange.

V. CONCLUSION

The predominance of matter over antimatter in the universe can be produced CP-violation in the decays of heavy neutrinos followed by sphaleron processes that recycle the resulting lepton asymmetry into a baryon asymmetry. We have shown that if the fermion mass matrices are determined by imposing an Abelian family symmetry then there are simple order-of-magnitude estimates of the CP-violation parameter ϵ_1 and the dilution mass $\tilde{m}_1$ that are critically important for determining the size of the lepton asymmetry produced in the early universe. In the most straightforward case these parameters are given by universal formulas in terms of the $U(1)$ quantum numbers ($\epsilon_1 \sim (3/16\pi)\lambda^{2(\mathcal{N}_1+L_3)}$ and $\tilde{m}_1 \sim \lambda^{2L_3} v_2^2/\Lambda_L$), and $\tilde{m}_1$ can be simply related to the experimentally determined light neutrino masses.

A Z_2 horizontal symmetry can be employed to reconcile (a) the large mixing that must be present to explain the atmospheric neutrino data with (b) a hierarchy in neutrino masses. We have shown here that employing this same Z_2 horizontal symmetry can enhance the

lepton asymmetry that results from heavy right-handed neutrino decays. This results in an enhanced baryon asymmetry in the universe. The change in the generated lepton asymmetry comes about because when a Yukawa coupling $(h_\nu^\dagger h_\nu)_{11}$ can be suppressed or enhanced compared to the usual expectation when the horizontal symmetry is $U(1)$. This affects the decay rate of the lightest Majorana neutrino N_1, as well as the amount of subsequent dilution of the asymmetry by lepton number violating scattering. A particular example where the generated asymmetry was explicitly calculated using the supersymmetric Boltzmann equations was given, and an enhancement of the lepton asymmetry (and hence ultimately the baryon asymmetry) by a factor seven was derived quantitatively.

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